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The influence of Hall effect on thermosolutal instability of a composite rotating plasma with finite Larmor radius

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Abstract The thermosolutal instability of a composite plasma is studied to include the Coriolis force, the finiteness of ion Larmor radius, collisions between ionized and neutral particles and Hall currents in the presence of a uniform magnetic field. It is found that, in the stationary convection case, the F. L. R. (finite Larmor radius), rotation, Hall currents (for high, values of magnetic and rotation parameters) and stable solute gradient have stabilizing effect on the system. However, the mutual collisions between ionized and neutral particles have no effect on stationary convection

Keywords : Thermosolutal instability, composite rotating plasma, Hall effect

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1. Introduction

The problem of the onset of thermal instability in the presence of solute gradient is of great importance because of its applications in astrophysics and atmospheric physics (especially in ionosphere and solar atmosphere). In such cases, the buoyancy forces can be caused not only by density differences due to the variations in temperature (according to the theory of thermal instability of a fluid layer heated from below), but also from those due to the variations in solute concentration.

The thermal instability of a composite plasma with finite electrical conductivity, in the absence/presence of Hall effect was studied by Sharma and Sharma [1,2], Gupta [3] and Vasiu and Beu [4]. The same problem, but taking into account the effects induced by the presence of compressibility, Hall currents and porous media, has been studied by Sharma and Rani [5] and Sharma and Sunil [6, 7].

In the stellar case, where the physics is quite similar to the thermohaline configuration, the finite Larmor radius, rotation and collisional effects are likely to be important. Vasiu and Marcu [8] studied the effects induced by the simultaneous presence of rotation, collisions between ionized and neutral particles, and F. L. R effect on the thermosolutal instability of a composite plasma, proving their stabilizing influence.

The present paper extends this problem to include the effect of Hall currents.

Here, we consider the thermosolutal instability of a composite incompressible plasma in rotation with a uniform angular velocity $\boldsymbol{\Omega}\,(0, 0, \Omega)$, subjected to a vertical magnetic field $\boldsymbol{B}\,(0, 0, B_0)$. The plasma is confined in the form of infinite horizontal layer of thickness l_0 and is acted upon by the vertically downward gravitational acceleration $\boldsymbol{g}\,(0, 0, -g)$. This plasma layer has two incompressible components : an ionized one and a neutral one with densities ρ_i and ρ_n respectively. The frequency of collision between the ionized and neutral particles is denoted by ν_c. We have neglected the influence of rotational motion and viscosity on the neutral plasma component. The effect of F. L. R on ionized component requires that the pressure must be a tensor quantity depending on ion gyration frequency, because of strong magnetic field action. Furthermore, the effects of viscosity and finite electrical conductivity of ionized component should also be considered. The plasma layer is heated from below and is subjected to a stable solute gradient. We have denoted the uniform temperature and uniform solute gradients by $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ and $\beta'\left(=\left|\frac{dC}{dz}\right|\right)$, respectively. In stationary state, the plasma layer gives $T = T_0 - \beta\,.\,z$ and $C' = C_0 - \beta'.z$ and $\rho = \rho_0\,[\,1 + \alpha(T_0 - T) - \alpha'(C_0 - C)] = \rho_0\,(1 + \alpha\;\beta\,.\,z - \alpha'\,.\,\beta'\,.\,z)$, where T_0, C_0 and T, C are the temperatures and concentrations at the bottom surface $z = 0$ and at any point between $z = 0$ and $z = l_0$, z-axis being taken as the vertical axis, ρ_0 is the density at $z = 0$; α, α' represent the thermal coefficient of expansion and solvent coefficient of expansion, respectively.

2. Linearized perturbation equations

We make the assumption that both incompressible viscous ionized fluid and incompressible nonviscous neutral gas behave like continuum fluids, and the neutral gas is not affected by the pressure gradient, gravitational acceleration, temperature gradient and stable solute gradient

On the basis of foregoing remarks and following the linearized perturbation theory [9], we start with the following linearized hydromagnetic equations of the system :

$$\frac{\partial}{\partial t}(\delta \boldsymbol{v}_i) = -\frac{1}{\rho_i}\nabla(\delta \boldsymbol{P}) + \nu_i \Delta(\delta \boldsymbol{v}_i) + \nu_c(\delta \boldsymbol{v}_n - \delta \boldsymbol{v}_i) - (\alpha\theta - \alpha'\gamma)\boldsymbol{g} +$$

$$+ 2\delta \boldsymbol{v}_i \times \boldsymbol{\Omega} + \frac{1}{\rho_i \mu_0}(\nabla \times \delta\boldsymbol{B}) \times \boldsymbol{B}, \tag{1}$$

$$\frac{\partial}{\partial t}(\delta \boldsymbol{v}_n) = -\frac{1}{\varepsilon}\nu_c(\delta \boldsymbol{v}_n - \delta \boldsymbol{v}_i), \tag{2}$$

$$\nabla \cdot (\delta \boldsymbol{v}_n) = 0, \tag{3}$$

$$\nabla \cdot (\delta \boldsymbol{v}_i) = 0, \tag{4}$$

$$\frac{\partial\theta}{\partial t} - \chi\Delta\theta = \beta\, w, \tag{5}$$

$$\frac{\partial\gamma}{\partial t} - \chi'\Delta\gamma = \beta'\, w, \tag{6}$$

$$\frac{\partial}{\partial t}(\delta \boldsymbol{B}) = \nabla \times (\delta \boldsymbol{v}_i \times \boldsymbol{B}) + \nu_m \Delta(\delta \boldsymbol{B}) - H_a \cdot \nabla \times [(\nabla \times \delta \boldsymbol{B}) \times \boldsymbol{B}], \tag{7}$$

$$\nabla \cdot (\delta \boldsymbol{B}) = 0, \tag{8}$$

where $\delta \boldsymbol{v}_i = (u_i, v_i, w_i)$, $\delta \boldsymbol{v}_n = (u_n, v_n, w_n)$, $\delta \boldsymbol{P}$, $\delta \boldsymbol{B}$ $(\delta B_x, \delta B_y, \delta B_z)$, θ, γ denote the perturbations in velocities $\boldsymbol{v}_i$, $\boldsymbol{v}_n$, the stress tensor $\boldsymbol{P}$, magnetic field $\boldsymbol{B}$, temperature T, concentration C ; where as χ, χ', ν_m, ν_i, ν_C are the thermal diffusivity, solute diffusivity, electrical resistivity, kinematic viscosity of ionized component and ion-neutral collisional frequency respectively ; $\varepsilon = \frac{\rho_n}{\rho_i}$; $H_a = \frac{1}{Ne\mu_0}$ where N is the ion number density, e is electrical charge of ions and μ_0 is the vacuum permeability.

The change in the density $\delta \rho$ is caused by the perturbation θ in the temperature and γ in the concentration, and is given by :

$$\delta \rho = -\rho_0 (\alpha \theta - \alpha' \gamma). \tag{9}$$

We make the assumption that both ionized fluid and neutral gas behave like continuum fluids and the influence of magnetic field is negligible for the neutral gas. For the vertical magnetic field $\boldsymbol{B}$ $(0, 0, B_0)$, the perturbations $\delta \boldsymbol{P}$ in the stress tensor $\boldsymbol{P}$ have the components as given below :

$$\delta P_{11} = \delta P_{xx} = \delta P - \rho_i \nu_0 \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right.$$

$$\delta P_{12} = \delta P_{21} = \delta P_{xy} = \delta P_{yx} = \delta P + \rho_i \nu_0 \quad \partial u \quad \partial v$$

$$\delta P_{22} = \delta P_{yy} = \delta P + \rho_i \nu_0 \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \tag{10}$$

$$\delta P_{13} = \delta P_{31} = \delta P_{xz} = \delta P_{zx} = \delta P - 2\rho_i \nu_0 \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right.$$

$$\delta P_{23} = \delta P_{32} = \delta P_{yz} = \delta P_{zy} = \delta P + 2\rho_i \nu_0 \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right),$$

$$\delta P_{33} = \delta P_{zz} = \delta P .$$

Here, δP is the perturbation in scalar pressure, $\rho_i \nu_0 = \dfrac{N_i T_i}{4\omega}$ where ω_i is the ion gyration frequency, ν_0 is the ion gyro-viscosity, N_i is the density number and T_i is the temperature of ions.

The perturbation $\delta\varphi$ in the terms of the normal mode has the form :

$$\delta\varphi(x, y, z, t) = \varphi^*(z) \cdot \exp[ik_x x + ik_y y + nt], \tag{11}$$

where φ^* is the amplitude, k_x, k_y are the wave numbers along x and y directions, $k^2 = k_x^2 + k_y^2$ and n is the growth rate, which is a complex constant.

Making use of (ii), eq. (2) yields

$$\Omega_n \, \delta v_n = \frac{1}{\varepsilon} v_C \, \delta v_i \, , \tag{12}$$

where $\Omega_n = n + \dfrac{v_C}{\varepsilon}$

Eq. (1) using (11-12) takes the form :

$$\Omega_{in} \, \delta v_i = -\frac{1}{\rho_i} \nabla(\delta P) - (\alpha\theta - \alpha'\gamma) \boldsymbol{g} + 2\delta v_i \times \Omega + \frac{1}{\mu_0 \rho_i} (\nabla \times \delta \boldsymbol{B}) \times \boldsymbol{B}, \tag{13}$$

where $\Omega_{in} = n^* - v_i \, \Delta ; n^* = n(1 + \varepsilon v_C / (n\varepsilon + v_C))$.

Using (10) and applying the "curl" operator on (13) we can obtain its projection along z-axis :

$$\Omega_{in} \zeta = [\, v_0 \, (2D^2 + k^2) + 2\Omega] \, Dw + \frac{B_0}{\mu_0 \rho_i} \, D\xi, \tag{14}$$

where $D = \dfrac{d}{dz}$, $D^2 = \dfrac{d^2}{dz^2}$, $\zeta = [\nabla \times (\delta v_i)]_z$, $\xi = [\nabla \times (\delta \boldsymbol{B})]_z$.

Using (11) and $[\nabla \times (\delta v_i \times B)]_z = B_0 D w_i$, the projection along the z-axis of (7) can be reduced to the following form :

$$\Omega_m \, \delta B_z = B_0 \, Dw - H_a B_0 \, D\xi, \tag{15}$$

where $\Omega_m = n + v_m \, (k^2 - D^2) = n - v_m \Delta$.

Application of the "curl (curl)" operator on (13) and "curl" on (17) results in their projections along z-axis :

$$\Omega_{in} \Delta \, w_i = \alpha \, g \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) - \alpha' \, g \left(\frac{\partial^2 \gamma}{\partial x^2} + \frac{\partial^2 \gamma}{\partial y^2} \right)$$

$$-[v_0 (2D^2 + k^2) + 2\Omega] \, D\zeta + \frac{B_0}{\mu_0 \rho_i} D\Delta\delta B_z \, , \tag{16}$$

$$\Omega_m \xi = B_0 \, D\zeta + H_a \, B_0 \, D\Delta(\delta B_z), \tag{17}$$

where we have used the following relations :

$$-H_a \nabla \times \{\nabla \times [(\nabla \times \delta \boldsymbol{B}) \times \boldsymbol{B}]\} = -H_a B_0 D[\nabla \times (\nabla \times \delta \boldsymbol{B})] = H_a B_0 D\Delta(\delta \boldsymbol{B}),$$

$$\nabla \cdot (\delta \boldsymbol{B}) = 0 .$$

Analyzing the disturbances in terms of normal modes, we assume that the perturbation quantities are of the form :

$$\{w_i, \theta, \gamma, \delta B_z, \zeta, \xi\} = \{W(z), \Theta(z), \Gamma(z), K(z), Z(z), X(z)\} \cdot \exp[ik_x x + ik_y y + nt], \tag{18}$$

where $W, \Theta, \Gamma, K, Z, X$ are the perturbation amplitudes.

It is convenient to discuss eqs. (5), (6) (14-17) in dimensionless variables and to take into account (18). We now choose the unit of length $[L] = l_0$ and of time $[T] = \frac{l_0^2}{\nu}$ and let $z^* = l_0 z\,;\ a = k\,l_0\,;\sigma = \frac{n l_0^2}{\nu}\,;\ \sigma^* = \frac{n^* l_0^2}{\nu}\,;p_1 = \frac{\nu}{\chi}\,;p_2 = \frac{\nu}{\nu_m}\,;p_3 = \frac{\nu}{\chi'}$. where $\nu = \nu_i$.

Introducing the following quantities :

$$C_0 = \frac{\beta l_0^2}{\chi};\ C_1 = \frac{\beta' l_0^2}{\chi'};\ C_2 = \frac{\sqrt{\bar{T}}}{l_0};\ C_3 = \frac{B_0\, l_0}{\mu_0 \rho_i \nu};$$

$$C_4 = \frac{\sqrt{U}}{l_0};\ C_5 = \frac{B_0\, l_0}{\nu_m};\ C_6 = \frac{\alpha g\, l_0^2}{\nu}a^2\,;\ C_7 = \frac{\alpha' g\, l_0^2}{\nu}a^2\,; \tag{19}$$

$$C_8 = \sqrt{U}\, l_0\,;\ C_9 = \sqrt{\bar{T}}\, l_0\,;\ \sqrt{\bar{T}} = 2\Omega\frac{l_0^2}{\nu};\ \sqrt{U} = \frac{\nu_0}{\nu};$$

$$C_{10} = \frac{H_a B_0 l_0}{\nu_m};\ C_{11} = \frac{H_a\, B_0}{\nu_m l_0}$$

and the operators

$$D = \frac{1}{l_0}\cdot\frac{d}{dz};O = D^2 - a^2\,;O^*_\sigma = D^2 - a^2 - \sigma^*\,;O_1 = D^2 - a^2 - \sigma p_1\,; \tag{20}$$

$$O_2 = D^2 - a^2 - \sigma p_2\,;O_3 = D^2 - a^2 - \sigma p_3\,;O_a = 2D^2 + a^2,$$

We obtain the final form of the aforesaid equations :

$$O_1\Theta = -C_0 W, \tag{21}$$

$$O_3\Gamma = -C_1 W, \tag{22}$$

$$O_\sigma{}^* Z = -[C_2 + C_4 O_a]DW - C_3 DX, \tag{23}$$

$$O_2 K = -C_5 DW + C_{10} DX, \tag{24}$$

$$O_\sigma{}^* OW = C_6\Theta - C_7\Gamma + [C_8 O_a + C_9]DZ - C_3 ODK, \tag{25}$$

$$O_2 X = -C_5 DZ - C_{11} ODK. \tag{26}$$

3. Dispersion equation

We now introduce the Chandrasekhar number $Q = C_3 C_5 = \frac{B_0 l_0}{\mu_0 \rho_i \nu \nu_m}$ -, $M = \frac{H_a^2\, B_0^2}{\nu_i}$ dimensionless number characterising Hall effect, and some new operators, *e.g.*,

$$L_M = O_2{}^2 + MOD^2;\ L^* = O_2 O_\sigma{}^* - QD^2\,;\ L = L^* L_M + QMOD^4.$$

Eliminating $\Theta(z), \Gamma(z), K(z), X(z)$ and $Z(z)$ from the eqs. (21-26) we obtain :

$$LK = -C_5 L^* O_2 DW + C_5 C_{10}(C_2 + C_4 O_a) O_2 D^3 W \tag{27}$$

and

$$LZ = -[C_2 + C_4 O_a] L_M O_2 DW - C_3 C_5 C_{11} O O_2 D^3 W. \tag{28}$$

Applying the operator $O_1 O_3 L$ in eq. (25) and using eqs. (21-24), (26), (27) and (28), we finally obtain the dispersion equation :

$$\{O O_1 O_3 [L O_\sigma^* - Q D^2 O_2 L^*] + [\tilde{T} + 2\sqrt{U\tilde{T}} O_a + U O_a^2] O_1 O_2 O_3 L_M D^2 +$$
$$+ 2Q[\sqrt{\tilde{T}M} + \sqrt{UM} O_a]\ O O_1 O_2 O_3 D^4\} W = (-R O_3 + S O_1) a^2 L W, \tag{29}$$

where

$R = \dfrac{g\,\alpha\,\beta\, l_0^4}{\chi\,\nu}$ is the Rayleigh number,

$S = \dfrac{g\alpha'\beta' l_0^4}{\chi'\nu}$ is the solute Rayleigh number,

$V = \dfrac{2\Omega l_0^2}{\nu_0}$ is the non-dimensional number accounting for F.L.R. and rotational effects

and $V\sqrt{U} = \sqrt{\tilde{T}}$

4. Particular cases

1. In the absence of Hall effect ($H_a = 0$, $M = 0$) eq. (29) reduces to :

$$(O_2 O_\sigma^* - QD^2)\{O O_1 O_3 (O_2 O_\sigma^* - QD^2) + O_2 a^2 (R O_3 - S O_1)\} W(z)$$
$$= [U(V + O_a)^2 O_1 O_3 O_2^2 D^2] W(z), \tag{30}$$

where $U(V + O_a)^2 = U(V^2 + 2VO_a + O_a^2) = \tilde{T} + 2\sqrt{U\tilde{T}} O_a + U O_a^2$, $UV^2 = \tilde{T}$.

Eq. (30) is identical to that in Vasiu and Marcu [8].

2. In the absence of F.L.R. ($\nu_0 = 0$, $U = 0$) and rotational motion ($\Omega = 0$, $\tilde{T} = 0$) eq. (29) becomes :

$$O O_1 O_3 [O_\sigma^{*2} O_2^2 - QD^2 (2 O_2 O_\sigma^* - QD^2)] W(z) + M O^2 O_\sigma^{*2} O_1 O_3 D^2 W(z)$$
$$= [(-R O_3 + S O_1) . a^2 (O_2^2 O_\sigma^* - Q O_2 D^2 + M O D_\sigma^* D^2)] W(z). \tag{31}$$

Eq. (31) is identical to the one as given by Sharma and Sharma [1] for $O_\sigma^* = O_\sigma = D^2 - a^2 - \sigma$, where $\sigma^* = \sigma$ ($n^* = n$ for $\varepsilon_c = 0$).

3. For a single plasma component (pure plasma) in the absence of ion-neutral collisional frequency ($\nu_C = 0$, $\sigma^* = \sigma$, $n^* = n$), of F.L.R ($\nu_0 = 0$, $U = 0$), Hall effect ($H_a = 0$, $M = 0$) and solute gradient ($\beta' = 0$, $\alpha' = 0$, $S = 0$) eq. (29) has the following form :

$$O_1 \{O(O_2 - Q_\sigma D^2)^2 + \tilde{T} Q_2^2 D^2\} W(z) = -R a^2 O_2 (O_2 O_\sigma - QD^2) W(z), \tag{32}$$ (32)

which is identical with Chandrasekhar's result [9].

5. Discussion

In the case of stationary convection ($n = \sigma = n^* = \sigma^* = 0$) the forms of O_1, O_2, O_3 and O^*_σ, L^*, L_M operators are :

$$O_1 = O_2 = O_3 = O_\sigma^* = O = D^2 - a^2 \,;\; L^* = O^2 - QD^2 \,;\; L_M = O^2 + MOD^2 \,;$$
$$L = (O^2 - QD^2)(O^2 + MOD^2) + MQOD^4 . \tag{33}$$

Eq. (29) can be reduced to :

$$O\{[O^2(O^2 + MOD^2 - QD^2) - QD^2(O^2 - QD^2)] + U(V + O_a)^2(O + MD^2)D^2$$
$$+2Q(\sqrt{\tilde{T}M} + \sqrt{UMO_a})D^4\}W(z) = (-Ra^2 + Sa^2)(O^2 + MOD^2 - QD^2)W(z). \tag{34}$$

In the absence of F.L.R ($v_0 = 0$, $U = 0$) eq. (34) becomes :

$$O\{O^2[O^2 + MOD^2 - QD^2] - Q(O^2 - QD^2)D^2 + 2Q(\sqrt{\tilde{T}M}D^4$$
$$+\tilde{T}(O + MD^2)D^2\}W(z) = (-Ra^2 + Sa^2)(O^2 + MOD^2 - QD^2)W(z). \tag{35}$$

The boundary conditions in the case in which both boundaries are free as well as perfect conductors are :

$$W(z) = D^2W(z) = DZ(z)\,0 \,;\; \Theta(z) = \Gamma(z) = X(z) = 0$$

at $z = 0$ and $z = 1$ ($z^* = 0, z^* = l_0$) and δB_x, δB_y, δB_z are continuous. The proper solution of eq. (34) characterizing the lowest mode has the form :

$$W(z) = W_0 \sin(\pi z). \tag{36}$$

where W_0 is constant.

Substituting (36) in (34) we obtain the characteristic equation :

$$R_1 = \left(\frac{1+x}{x}\right)\left\{\frac{[(1+x)^2 + Q_1]^2 + M(1+x)^3 + U[V_1 - (2-x)]^2(1+M+x)}{(1+x)^2 + M(1+x) + Q_1}\right.$$
$$\left. + \frac{2Q[\sqrt{\tilde{T}M} + \sqrt{UM}(2-x)]}{(1+x)^2 + M(1+x) + Q_1}\right\} + S_1 . \tag{37}$$

where $x = \dfrac{a^2}{\pi^2}$; $R_1 = \dfrac{R}{\pi^4}$; $S_1 = \dfrac{S}{\pi^4}$; $Q_1 = \dfrac{Q}{\pi^2}$; $V_1 = \dfrac{V}{\pi^2}$; $\tilde{T}_1 = \dfrac{\tilde{T}}{\pi^4}$.

In the absence of Hall effect ($Ha = 0$, $M = 0$) eq. (37) can be reduced to the form obtained by Gupta and Singh [10].

In the absence of F. L. R ($v_0 = 0$, $U = 0$) and rotational motion ($\Omega = 0$, $V = 0$, $\tilde{T} = 0$) eq. (37) is identical with Sharma and Sharma result [1].

Our discussion is limited only to the stationary convection (see eq. 37) where the modified Rayleigh number R_1 attains the minimum when $dR_1/dx = 0$. We obtain :

$$x^7 + a_1x^6 + a_2x^5 + a_3x^4 + a_4x^3 + a_5x^2 + a_6x + a_7 = 0 \tag{38}$$

where
$$a_1 = 0.5(11 + 4M + U),$$
$$a_2 = M^2 + M(U+9) + 2(Q_1 + U) + 12,$$
$$a_3 = 0.5[M^2(U+7) + M(5Q_1 + 6U + 30) + Q_1(3U + 13) + U(2 + 4V_1 - V_1^2)$$
$$+2Q_1\sqrt{UM} + 25],$$
$$a_4 = M^2(U+4) + M[(Q_1(6+U) - U(V_1^2 - 4V_1 + 1) + 10] + Q_1^2$$
$$+2Q_1[U(V_1 - 1) + 3] - U[2V_1^2 - 8V_1 + 6) - 2Q_1(\sqrt{MU} + \sqrt{\tilde{T}_1 M} + 5,$$
$$a_5 = 0.5\{M^2[2 + U(2V_1 - V_1^2 - 3)] + M[-Q_1^2 + Q_1(2UV_1 - 3U + 6)$$
$$-2UV_1(3V_1 - 12) - 22U] + Q_1^2 + Q_1(UV_1^2 - 3U + 2) + U(24V_1 - 6V_1^2 - 23)$$
$$-2MQ_1(\sqrt{M\tilde{T}_1} + 2\sqrt{MU}) - Q_1(18\sqrt{MU} + 10\sqrt{M\tilde{T}_1}) - 2Q_1^2\sqrt{MU} - 3\} \tag{39}$$
$$a_6 = -\{M^2[1 + U(V_1 - 2)^2] + M[Q_1^2 + 2Q_1 + 2Q_1\sqrt{MT_1} + 4Q_1\sqrt{MU}$$
$$+3U(V_1 - 2)^2 + 3] + 2[Q_1^2 + 2Q_1 + U(V_1 - 2)^2 + 4Q_1\sqrt{MU} + 2Q_1\sqrt{M\tilde{T}_1} + 11\},$$
$$a_7 = -0.5\{M^2[1 + U(V_1 - 2)^2] + M[Q_1^2 + Q_1(U(V_1 - 2)^2 + 2)$$
$$+2U(V_1 - 2)^2 + 2] + (Q_1 + 1)[(Q_1 + 1)^2 + U(V_1 - 2)^2]$$
$$+2MQ_1(M + Q_1)(2\sqrt{MU} + \sqrt{M\tilde{T}_1})\},$$

with x_l ($l = 1, 7$) determined as a solution of eq. (38), the relation (37) will give the required critical Rayleigh number R_c (if $R < R_c$ the system is stable and for $R > R_c$ the system is unstable)

The investigation of F.L.R., rotational motion, solute gradient and Hall effects is facilitated by the analytical study of dR_1/dU, dR_1/dV_1, dR_1/dS_1, dR_1/dM. It follows from (37) that

$$\frac{dR_1}{dU} = \frac{\sqrt{U}[V_1 - (2-x)]^2 \cdot (1 + x + M)\cdot(1+x) + Q_1(1+x)(2-x)\sqrt{M}}{x[(1+x)^2 + M(1+x) + Q_1]\sqrt{U}}$$

$$\frac{dR_1}{dV_1} \quad \frac{2U(1+x)(1+x+M)(V_1 - 2 + x)}{x[(1+x)^2 + M(1+x) + Q_1]}$$

$$\frac{dR_1}{dM} = \frac{(1+x)Q_1}{x[(1+x)^2 + M(1+x) + Q_1]^2\sqrt{M}}\{[(1+x)^2 + Q_1][\sqrt{\tilde{T}_1} + (2-x)\sqrt{U}$$
$$-(1+x)\sqrt{M}] + U\sqrt{M}[V_1 - (2-x)]^2 - M(1+x)[\sqrt{\tilde{T}_1} + (2-x)\sqrt{U}]\}, \tag{40}$$

$$\frac{dR_1}{dS_1} = 1$$

The numerical results and comparatively graphic analysis for our model can be seen in Figures 1-4. In Figure 1, we have plotted the variation of R_1 with x for fixed $Q_1 = 10$, $\tilde{T}_1 = 10^2$, $U = 10^2$, $S_1 = 10^3$ and for various values of $M = 5, 50, 500$. Each of these curves shows a minimum and its found that for a fixed value of x, $(R_1)_{min}$ decreases with increase in M, this demonstrates the destabilizing influence of Hall currents for small values of Q_1 and $\tilde{T}_1$. In this case of a small magnetic and rotation parameters, the presence of Hall current induces a vertical component of vorticity and this may well be the reason for the destabilizing influence.

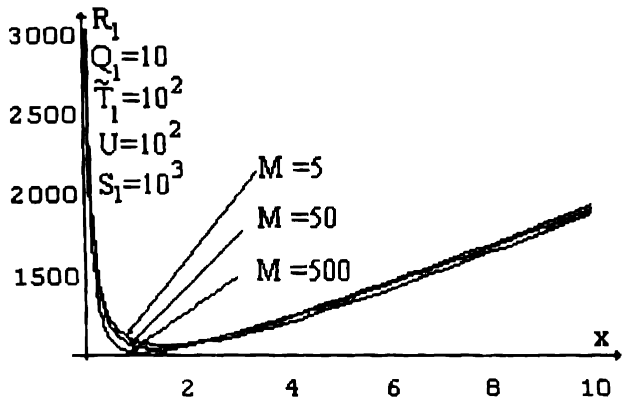

Figure 1. Variation of R_1 with x for a fixed $Q_1 = 10$, $\tilde{T}_1=10^2$, $U = 10^2$, $S_1 = 10^3$ and for various values of $M = 5, 50, 500$

The increase in Q_1 and $\tilde{T}_1$ (Figure 2) for the same values of M, shows the increase of $(R_1)_{min}$ with increase in M, proving the stabilizing influence of Hall currents in the presence of a strong magnetic field and high value of angular velocity.

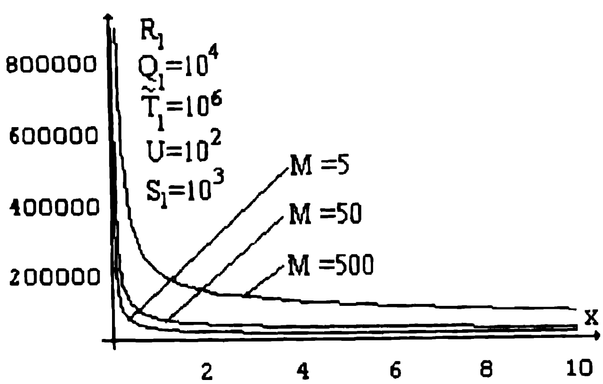

Figure 2. Variation of R_1 with x for a fixed $Q_1 = 10^4$, $\tilde{T}_1 = 10^6$, $U = 10^2$, $S_1 = 10^3$ and for various values of $M = 5, 50, 500$.

The relationship between Rayleigh number and x for three values of Taylor number $\tilde{T}_1$ $(= 10^4, 10^5, 10^6)$ with $Q_1 = 10^4$, $M = 50$, $U = 10^2$, $S_1 = 10^3$ is plotted in Figure 3. It is evident that for a fixed value of x, R_1 increase with the increase in $\tilde{T}_1$, and so $(R_1)_{min}$ increases with $\tilde{T}_1$, proving a stabilizing influence of the rotation in the presence of magnetic field, Hall currents, solute gradient and F. L. R effect.

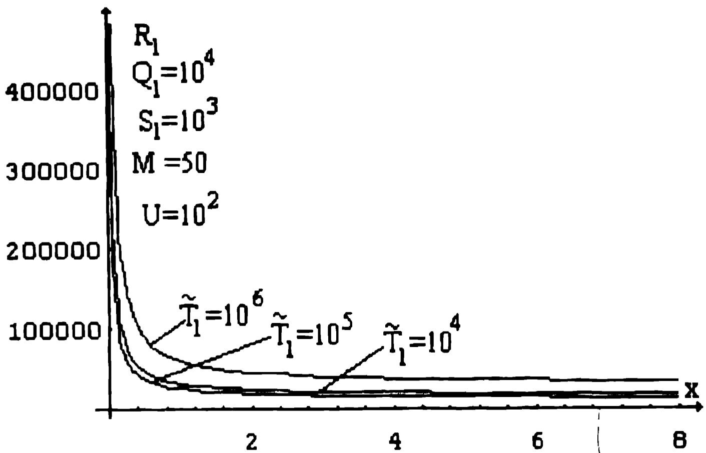

Figure 3. Variation of R_1 with x for a fixed $Q_1 = 10^4$, $M = 50$, $U = 10^2$, $S_1 = 10^3$ and for various values of $\tilde{T}_1 = 10^4, 10^5, 10^6$.

The relationship between R_1 and x for three values of U (= 1, 10, 100) parameter with $Q_1 = 10$, $M = 5$, $\tilde{T}_1 = 10^2$, $S_1 = 10^3$ (Figure 4), illustrates the increase of R_1 with U. This demonstrates the stabilizing effect of F. L. R.

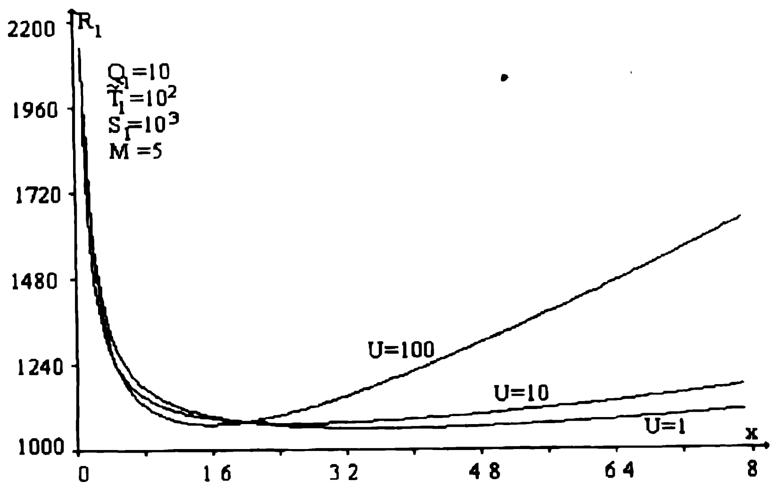

Figure 4. Variation of R_1 with x for a fixed $Q_1 = 10$, $M = 5$, $\tilde{T}_1=10^2$, $S_1 = 10^3$ and for various values of U = 1,10,100.

It has been shown, in the stationary convection case, that the Hall currents (for high values of magnetic and rotation parameters), F. L. R., rotation and stable solute gradient have stabilizing effect on the system while the mutual collisions between ionized and neutral particles have no effect on stationary convection.

stabilizing effect on the system while the mutual collisions between ionized and neutral particles have no effect on stationary convection.

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